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necessary to find a series which will permit the two sets of La Hirian primaries to be each divided into three groups of numbers having a common summation, as exemplified in the arithmetical series 1, 2, 3, 4......81. In this series the two sets of La Hirian primaries are

$$\begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 0 & 9 & 18 & 27 & 36 & 45 & 54 & 63 & 72 \end{array}$$

and the above may be rearranged in three groups as follows:

$$\begin{array}{ccc|ccc|ccc} 1 & 5 & 9 & 3 & 4 & 8 & 2 & 6 & 7 \\ 0 & 36 & 72 & 18 & 27 & 63 & 9 & 45 & 54 \end{array}$$

in which each triplet in the upper line sums 15 and each triplet in the lower line sums 108. The difficulty of finding a 9^2 series of prime numbers that will meet the above conditions appears to be insurmountable.

The writer believes the squares in this paper to be of the lowest possible summations, but no claim to that effect is made except in the case of the 4^2, though it is probable that 5^2 and 7^2 are also of minimum summation.

It is hoped that some student of magic squares may be able before long to make a pandiagonal square of the 9th order with prime numbers.

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Wyoming N. J.

PANELED MAGIC SQUARES.*

These squares are made with a central magic square having one or more panels or borders of figures, so arranged that each enlargement forms another magic square.

Paneled magic squares may be either "perfect" or "imperfect," the former being those in which all intermediate squares are magic, and the latter those wherein one or more of the intermediate squares are not magic.

These two varieties are constructed by different rules. The "perfect" squares are formed entirely of couplets, with the exception of the center cells of odd squares, and the inner 4^2 of even squares, while "imperfect" squares may be made with couplets, triplets, quadruplets, etc.

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* Diagrams by Mr. H. A. Sayles, Schenectady, N. Y.

Only perfect paneled squares will be considered in this article and these will be divided into two classes wherein n is respectively odd and even. Furthermore, only prime numbers will be used in their construction.

PANELED MAGIC SQUARES IN WHICH n IS ODD.

In constructing squares of this class of all orders, it is necessary to find

$$4+8+12 \ldots\ldots\ldots\ldots + 2(n-1)$$

couplets, each summing twice a certain prime number which must occupy the center cell of the square. The prime numbers forming these couplets must also contain a 3^2 series answering to the following formula:

$$\begin{array}{lll} a & a+b & a+2b \\ a+c & a+b+c & a+2b+c \\ a+2c & a+b+2c & a+2b+2c \end{array}$$

After the 3^2 is made from this series the remaining couplets must then be arranged in magic order.

Paneled Magic Square of the 5th Order.

It is obvious that this is the smallest possible paneled square, and it must be formed of a magic 3^2 which is part of and entirely enclosed in a magic 5^2. Twelve couplets must be used in its construction, each couplet summing twice the prime number which is to occupy the center cell. Twelve such couplets can be found, each summing 466, but as 3 is one of the constituent numbers, it can be easily proved by analysis of the congruences (mod. 3) that it is impossible to make a magic square with these numbers. In the next higher available series, each couplet sums 502, and a paneled magic

401	11	263	149	431
419	461	23	269	83
311	59	251	443	191
53	233	479	41	449
71	491	239	353	101

Fig. 1.

square made with these couplets is shown in Fig. 1. In the 5^2, S = 1255 and in the 3^2, S = 753.

Paneled Magic Square of the 7th Order.

For this square a series of 24 couplets is required, and 617 × 2 = 1234 is the lowest number which will permit this series of couplets. This series, however, contains a 3 and is therefore inadmissible.

461	1193	311	419	1229	353	521
1151	1277	1049	173	443	263	131
1013	59	1181	23	719	1223	269
191	599	179	641	1103	683	1091
509	251	563	1259	101	1031	773
401	1019	233	1109	839	5	881
761	89	971	863	53	929	821

Fig. 2.

The next series sums 641 × 2 = 1282, and Fig. 2 shows a 7^2 which has been made with these couplets, the different summations being, $7^2 = 4487$, $5^2 = 3205$ and $3^2 = 1923$.

Paneled Magic Square of the 9th Order.

This square demands 40 couplets, each summing twice the prime number which must occupy the center cell. Two sets of 40 couplets can be found summing 2554 and 2722 respectively, but as each series contains a 3, neither is admissible. There are 41 couplets

2087	2441	2477	191	887	863	857	593	1907
2267	1163	2711	1451	2423	71	1097	653	467
2381	2633	41	2729	1553	1361	1151	101	353
2351	1721	2687	2273	401	1427	47	1013	383
317	113	1223	521	1367	2213	1511	2621	2417
491	947	1301	1307	2333	461	1433	1787	2243
761	911	1583	5	1181	1373	2693	1823	1973
821	2081	23	1283	311	2663	1637	1571	1913
827	293	257	2543	1847	1871	1877	2141	647

Fig. 3.

summing 2734, one of which contains a 3, and a paneled magic square constructed from the remaining 40 couplets is shown in Fig. 3.

The 3^2 sums 4101, the 5^2 6835, the 7^2 9569 and the 9^2 12303, respectively.

PANELED MAGIC SQUARES IN WHICH n IS EVEN.

To construct these squares

$$10+14+18.........+2(n-1)$$

couplets are required. These couplets need not sum twice a prime number as in squares where n is odd. The panels are made as in the odd squares and the inner 4^2 is formed from such primes as remain over. The summation of the inner 4^2 must be twice that of the couplets.

Paneled Magic Square of the 6th Order.

Ten couplets are required to construct this square. Eleven couplets of prime numbers may be selected summing 144, but a 4^2 summing 288 has not yet been found, and it is doubtful if it can be made from the series of prime numbers remaining after the outside panel is finished. The next lowest summation for a series of ten couplets is 150, and a square constructed with these couplets is shown in Fig. 4, wherein the 6^2 has S = 450 and the 4^2 has S = 300.

97	43	139	1	67	103
61	31	71	7	191	89
23	13	167	101	19	127
109	251	3	29	17	41
113	5	59	163	73	37
47	107	11	149	83	53

Fig. 4.

Paneled Magic Square of the 8th Order.

Twenty-four couplets are necessary for this square, and these can be found summing 330, but they do not appear to contain a 4^2 series. The lowest possible summation for these couplets containing a 4^2 series is 390, and the square made with them is shown in Fig. 5, wherein the 8^2 sums 1560, the 6^2 1170, and the 4^2 780.

Paneled Magic Square of the 10th Order.

Forty-two couplets are needed for this square, and the lowest possible summation for same is found to be 660. Fig. 6 shows a square made with these couplets, the various summations being, $10^2 = 3300$, $8^2 = 2640$, $6^2 = 1980$ and $4^2 = 1320$.

379	233	17	151	227	37	149	367
127	197	389	277	7	109	191	263
59	41	3	179	107	491	349	331
139	79	467	269	13	31	311	251
317	347	53	19	479	229	43	73
293	307	257	313	181	29	83	97
223	199	1	113	383	281	193	167
23	157	373	239	163	353	241	11

Fig. 5.

61	587	563	487	103	157	449	571	239	83
137	19	241	263	647	643	409	389	29	523
193	43	349	1	313	281	653	383	617	467
139	59	307	31	107	499	683	353	601	521
197	229	67	79	719	331	191	593	431	463
547	619	613	941	37	233	109	47	41	113
479	607	367	269	457	257	337	293	53	181
509	433	277	659	347	379	7	311	227	151
461	631	419	397	13	17	251	271	641	199
577	73	97	173	557	503	211	89	421	599

Fig. 6.

It will be seen that this brief paper covers all orders of paneled magic squares made with prime numbers up to the 10th order inclusive, and it is believed that all these squares show the lowest possible summations.

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